Workshop in Mathematical Imaging and Random Geometry

25-26 November 2024, Nice
Laboratoire Jean Alexandre Dieudonné

Local organising committee:
Michele Ancona, Elena Di Bernardino, Maxime Ingremeau, Khazhgali Kozhasov, and Radomyra Shevchenko

Program

Conference Room, Laboratory Jean Alexandre Dieudonné, Parc Valorse, Nice

Monday 25 November 2024

09h30-10h00 Welcome and coffee

10h00-11h00 Thibaut Le Gouic *"Sampler for the Wasserstein barycenter "*

11h00-12h00 Blanche Buet *"Multi-dimensional varifolds to handle discrete surfaces"*

Lunch

14h00-15h00 Thomas Letendre *"Finiteness of moments for the zeros of Gaussian fields"*

15h00-16h00 Hugo Vanneuville *"Noise sensitivity and percolation"*

Coffee break

16h30-17h30 Claire Launay *"Texture modeling : Fractional Brownian fields and monogenic images"*

17h30-18h15 Short talks

- Olivier Bisson *"Riemannian Metrics on Correlation Matrices with Applications in Neuroimaging"*
- Ryan Cotsakis *"The moments of the discretization error in perimeter estimation of excursion sets"*
- Émile Pierret *"Diffusion models for Gaussian distributions: Exact solutions and Wasserstein errors"*

19h30 Conference diner

Tuesday 26 November 2024

10h00-11h00 Anna Paola Todino *"Laguerre Expansion for Nodal Volumes"*

11h00-12h00 Arthur Leclaire *"Generative Models for Image Synthesis"*

Lunch

Abstracts

(in alphabetic order)

Speaker: Olivier Bisson
Affiliation: INRIA Sophia-Antipolis
Title: *Riemannian Metrics on Correlation Matrices with Applications in Neuroimaging*
Abstract: Symmetric positive definite matrices, particularly correlation matrices, have become a common source of geometric information. The challenges encountered with the conventional Euclidean metric have led to the development of alternatives based on Riemannian metrics. Correlation matrices are widely used to describe brain connectivity in

anatomical and functional neuroimaging. In this talk, we will first explore several methods to endow the space of correlation matrices with a differentiable structure. Next, we will present recent Riemannian metrics developed for full-rank correlation matrices and demonstrate how these metrics enable us to perform statistics on non-linear spaces. Finally, we will illustrate our work with applications in the study of brain connectomes, using data obtained from rs-fMRI.

Speaker: Blanche Buet
Affiliation: Laboratoire de Mathématiques d'Orsay
Title: *Multi-dimensional varifolds to handle discrete surfaces*
Abstract: We propose a natural framework for the study of surfaces and their different discretizations based on varifolds. Varifolds have been introduced by Almgren to carry out the study of minimal surfaces. Though mainly used in the context of rectifiable sets, they turn out to be well suited to the study of discrete type objects as well. While the structure of varifold is flexible enough to adapt to both regular and discrete objects, it allows to define variational notions of mean curvature and second fundamental form based on the divergence theorem. Thanks to a regularization of these weak formulations, we propose a notion of discrete curvature (actually a family of discrete curvatures associated with a regularization scale) relying only on the varifold structure. We performed numerical computations of mean curvature and Gaussian curvature on 3D point clouds to illustrate this approach. Though flexible, varifolds require the knowledge of the dimension of the shape to be considered. By interpreting the product of the Principal Component Analysis, that is the covariance matrix, as a sequence of nested subspaces naturally coming with weights according to the level of approximation they provide, we are able to embed all d-dimensional Grassmannians into a stratified space of covariance matrices. Building upon the proposed embedding of Grassmannians into the space of covariance matrices, we generalize the concept of varifolds to what we call flagfolds in order to model multi-dimensional shapes.

Speaker: Ryan Cotsakis
Affiliation: Expertise Center for Climate Extremes, Université de Lausanne
Title: *The moments of the discretization error in perimeter estimation of excursion sets*
Abstract: When the perimeter of a set with smooth boundary is inferred from a digital image, there is an induced discretaization error. In this talk, we will discuss the probabilistic properties of this error when estimating the perimeter of the excursion sets of C^2 stationary and isotropic random fields on $\mathbb{R}^d$. Our initial work focuses on the first moments of this error, and its invariance in the geometry of the discretization sites (when the excursion set is observed over a Voronoi diagram). We will see how the analysis may be pushed further and obtain expressions for the higher moments of the discretization error. In addition, we will see how these higher moments relate to the geometry of the observation sites, and how the regularity assumptions of the random field play a role in the expressions.

Speaker: Claire Launay
Affiliation: LMBA (Laboratoire de Mathématiques Bretagne Atlantique, UMR CNRS 6205), Université Bretagne Sud
Title: *Texture modeling : Fractional Brownian fields and monogenic images*
In this talk, we focus on specific anisotropic fractional Brownian fields (AFBFs) called lighthouse fields, whose self-similarity depends solely on the so-called Hurst parameter, while anisotropy is given by the opening angle of an oriented spectral cone. This fractional field generalizes fractional Brownian motion and models rough natural phenomena. Estimating

the model parameters is a crucial issue for modeling and analyzing various texture images. This work is a collaboration with Hermine Biermé, Philippe Carré and Céline Lacaux. It introduces the representation of AFBFs using the monogenic transform. Combined with a multiscale analysis, the monogenic signal is built from the Riesz transform to extract local orientation and structural information from an image at different scales. We then exploit the monogenic signal to define new estimators of AFBF parameters in the case of lighthouse fields. We prove that the estimators of anisotropy and the Hurst index are strongly consistent. We demonstrate that these estimators verify asymptotic normality with known variance. We also introduce an estimator of the texture orientation. Numerical results illustrate the performance of these estimators. Regarding Hurst index estimation, estimators based on the monogenic representation of random fields appear to be more robust than those using only the Riesz transform. We show that both estimation methods outperform standard estimation procedures in the isotropic case and provide excellent results for all degrees of anisotropy.

Speaker: Thibaut Le Gouic
Affiliation: Institut de Mathématiques de Marseille / Centrale Méditerranée
Title: *Sampler for the Wasserstein barycenter*
Abstract: Wasserstein barycenters have become a central object in applied optimal transport as a tool to summarize complex objects that can be represented as distributions. Such objects include posterior distributions in Bayesian statistics, functions in functional data analysis and images in graphics. In a nutshell a Wasserstein barycenter is a probability distribution that provides a compelling summary of a finite set of input distributions. While the question of computing Wasserstein barycenters has received significant attention, this talk focuses on a new and important question: sampling from a barycenter given a natural query access to the input distribution. We describe a new methodology built on the theory of Gradient flows over Wasserstein space. This is joint work with Chiheb Daaloul, Magali Tournus and Jacques Liandrat.

Speaker: Arthur Leclaire
Affiliation: Telecom Paris
Title : *Generative Models for Image Synthesis*
Abstract: The goal of this talk is to give an overview of several stochastic models that have been proposed for image synthesis, focusing first on texture synthesis and then on image synthesis from a database. In particular, we will show how to formulate the generative modeling problem by using optimal transport costs. This formulation leads to stochastic algorithms for learning neural networks for generative tasks, called Wasserstein generative adversarial networks. We will show how tools from optimal transportation can help to better understand the behavior of these algorithms. If time permits, the last part of the talk will introduce more recent generative models based on the reversion of a diffusion process.

Speaker: Thomas Letendre
Affiliation: Université Paris-Saclay
Title: *Finiteness of moments for the zeros of Gaussian fields*
Abstract: Let $f : \mathbb{R}^n \to \mathbb{R}$ be a centered non-degenerate Gaussian field, at least of class $\mathcal{C}^1$. Almost surely, its zero set Z is an hypersurface, equipped with its $(n-1)$-dimensional Hausdorff measure. In this talk, I will show that the volume of Z, and more generally its

linear statistics, have a finite p-th moment as soon as f is of class $\mathcal{C}^p$ and satisfies some higher order non-degeneracy condition.
Using Kac–Rice formulas, finiteness of this p-th moment is equivalent to the local integrability of some function on $(\mathbb{R}^n)^p$ which is singular along the diagonal. Proving this integrability turns out to be a geometric problem, fairly independent of the law of f. We solve it by using polynomial interpolation and some tools from algebraic geometry. This is a joint work with Michele Ancona.

Speaker: Émile Pierret
Affiliation: Université d'Orléans
Title: *Diffusion models for Gaussian distributions: Exact solutions and Wasserstein errors*
Abstract:
Diffusion or score-based models recently showed high performance in image generation. They rely on a forward and a backward stochastic differential equations (SDE). The sampling of a data distribution is achieved by solving numerically the backward SDE or its associated flow ODE. Studying the convergence of these models necessitates to control four different types of error: the initialization error, the truncation error, the discretization and the score approximation. In this paper, we study theoretically the behavior of diffusion models and their numerical implementation when the data distribution is Gaussian. In this restricted framework where the score function is a linear operator, we derive the analytical solutions of the backward SDE and the probability flow ODE. We prove that these solutions and their discretizations are all Gaussian processes, which allows us to compute exact Wasserstein errors induced by each error type for any sampling scheme. Monitoring convergence directly in the data space instead of relying on Inception features, our experiments show that the recommended numerical schemes from the diffusion models literature are also the best sampling schemes for Gaussian distributions.

Speaker: Anna Paola Todino
Affiliation: Università del Piemonte Orientale
Title: *Laguerre Expansion for Nodal Volumes*
Abstract: We investigate the nodal volume of random hyperspherical harmonics on the d-dimensional unit sphere. We exploit an orthogonal expansion in terms of Laguerre polynomials; this representation entails a drastic reduction in the computational complexity and allows to prove isotropy for chaotic components, an issue which was left open in the previous literature. As a further application, we obtain an upper bound (that we conjecture to be sharp) for the asymptotic variance, in the high-frequency limit, of the nodal volume for $d > 2$. This result shows that the so-called Berry's cancellation phenomenon holds in any dimension: namely, the nodal variance is one order of magnitude smaller than the variance of the volume of level sets at any non-zero threshold, in the high-energy limit. Joint work with Domenico Marinucci and Maurizia Rossi.

Speaker: Hugo Vanneuville
Affiliation: CNRS and Institut Fourier
Title: *Noise sensitivity and percolation*
Abstract: We say that a property of some object is noise sensitive if the knowledge of some noisy version of this object gives essentially no information about this property. In 1999, Benjamini, Kalai and Schramm proved that the large scale geometry of planar percolation is noise sensitive. With Vincent Tassion, we have proposed a new proof of this property.

This talk will be mainly an introduction to the theory of noise sensitivity, from a point of view which – we'll try – will be geometric. (And we will not assume that the audience is used to percolation models!)